Parametric tests

Josemari Sarasola

Statistics for Business

Parametric tests

Definition

Parametric tests are statistical tests about the value of parameter(s) of a distribution. E.g., $H_0 : \mu = 4$ (that is about the population mean) and $H_0 : p = 0.4$ (that is about the proportion of a population)

- It's absolutely wrong to state something like this: $H_0 : \hat{\mu} = 4$. That's because we can always know the value of an estimator from data, so it's no sense testing a value for it.
- Take a look at the last problem about CLT.

Parametric tests

Parametric tests and estimators

We apply estimators to perform statistical tests. E.g., taken $H_0 : \mu = 4$, if we get $\hat{\mu} = \overline{x} = 80$, we may claim that we have strong enough evidence to reject the null hypothesis, as it's amazing to have a sample mean of 80 (its probability is very small), when the population mean (μ) is 4.

Decision rule and test direction

Generally,

- We reject $H_0 : \theta \leq \theta_0$ when $\hat{\theta}$ (the corresponding estimate) is big enough (e.g., we reject $H_0 : \mu < \mu_0$ when $\overline{x}$ is bigger than the critical value $\overline{x}_0$). It's a one-sided test.
- We reject $H_0 : \theta \geq \theta_0$ when $\hat{\theta}$ (the corresponding estimate) is small enough (e.g., we reject $H_0 : \mu > \mu_0$ when $\overline{x}$ is smaller than the critical value $\overline{x}_0$). It's a one-sided test.
- We reject $H_0 : \theta = \theta_0$ when $\hat{\theta}$ (the corresponding estimate) is big enough as well as small enough (e.g., we reject $H_0 : \mu = \mu_0$ when $\overline{x}$ is larger than $\overline{x}_h$ and lower than $\overline{x}_t$). It's a two-sided test.

Remark: null hypothesis always include the equality sign.

Parametric tests

- In order to calculate the p-value for a given value of an estimator, we calculate the probability of that value or *something stranger*.
- We can calculate probabilities about estimators (arithmetic mean, median, ... from data) because they are actually random variables, that is to say, their value is random, because they are calculated from random samples.
- As the value of an estimator depends on a random sample, we call the probability distribution of an estimator a *sampling distribution*.
- Each estimator has its own(s) sampling distribution(s). Sampling distributions may vary according not only to the estimator, but also to the model and n sample size.
- Sampling distribution: in spanish, *distribución muestral*; in basque, *lagin banaketa*.
- Before learning the sampling distributions for common estimators, remember: μ is the population mean, and σ the population standard deviation.

$$\overline{x} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$$

$$t = \frac{\overline{x} - \mu}{\hat{s}/\sqrt{n}} \sim t_{n-1}$$

Remember the formula for sample (corrected) deviation:
$\hat{s} = \sqrt{\frac{\sum_i (x_i - \overline{x})^2}{n-1}}$.

Annex: Student's t distribution

Notation: $t \sim t_n$.

n are degrees of freedom (the parameter) and must be an integer value.

The more n is big, the bigger will the probability on the tails. For $n > 30$ it's almost equal to the normal distribution. As the normal distribution, it's symmetric around $x = 0$ axis.

$\chi^2_{1-\alpha,n}$ t_n $n = 1$ $n = 30$

Percentiles of Student's t distribution are tabulated when $n \leq 30$. The table gives percentiles (values corresponding for the the probability below) for probabilities larger than 0.5. For lower probabilities we apply symmetry.
For example:

- $t \sim t_4;\ P[t < t_0] = 0.99 \rightarrow t_0 = 3.75$
- $t \sim t_7;\ P[t < t_0] = 0.1 \rightarrow t_0 = -1.42$

When degrees of freedom are bigger than 30, we can apply $N(0,1)$ distribution as an approximation.
Irish chemist William Sealy Gosset found this distribution, as he was making reserach about small samples in a brewery. He published the results in 1908 with the Student alias.

We must have $n > 30$:

$$\overline{x} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$$

We must have $n > 30$:

$$\overline{x} \sim N(\mu, \frac{\hat{s}}{\sqrt{n}})$$

	σ known	σ unknown
Normal population	$\overline{x} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$	$t = \dfrac{\overline{x} - \mu}{\frac{\hat{s}}{\sqrt{n}}} \sim t_{n-1}$
Non-normal population, $n > 30$	$\overline{x} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$	$\overline{x} \sim N\left(\mu, \frac{\hat{s}}{\sqrt{n}}\right)$

We have 20 pieces in a sample. 4 of them are faulty. Sample proportion is $4/20{=}0.2$ and p is the *real* proportion in the population, usually unknown.
For $n > 30$ sample sizes:

$$\hat{p} \sim N\left(p, \sqrt{\frac{pq}{n}}\right)$$

We must have a normal population:

$$\frac{ns^2}{\sigma^2} \sim \chi^2_{n-1}$$

Remember:

- $s = \sqrt{\frac{\sum_i (x_i - \overline{x})^2}{n}} = \sqrt{\frac{\sum_i x_i^2}{n} - \overline{x}^2}$
- $\hat{s} = \sqrt{\frac{\sum_i (x_i - \overline{x})^2}{n-1}}$
- $\hat{s}^2 = \frac{n}{n-1} s^2$
- $s^2 = \frac{n-1}{n} \hat{s}^2$

Sampling distributions

We have seen that parameters always appear in sampling distributions. But we don't know the value for parameters.
How to set its value in order to specify the sampling distribution?
ANSWER: We take the value given by the null hypothesis.

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Methods to perform the test

- p-value: we calculate the probability of evidence (the estimate) or even more amazing (that's the p-value), and we compare it to α;
- critical value, we calculate the value in the sampling distribution corresponding to the α probability, in the correct direction, and so we set the critical region (the region whose values lead to reject H_0).

Parametric tests

One-sided and two-sided tests

- In one-sided tests the *amazing* thing is on one side.
- In two-sided tests the *amazing* thing is on both sides: so reference value for the probability is $\alpha/2$. We must compare p-value to $\alpha/2$.
- We can look at the null hypothesis to decide if the test is 1-sided or 2-sided

$$H_0 : \theta = \theta_0 \rightarrow \text{two-sided}$$

$$\left.\begin{array}{l} \mathrm{H}_0 : \theta \geq \theta_0 \\ \mathrm{H}_0 : \theta \leq \theta_0 \end{array}\right\} \rightarrow \text{one-sided}$$

Multiple hypothesis

- Hypothesis of this kind: $H_0 : \mu \geq 4$, $H_0 : \sigma^2 < 1$, are multiple hypothesis, because they include not an exact value, but several values.
- In those cases, which value should we take to perform the test?
- Answer: the boundary value. E.g., for $H_0 : \mu \geq 4$, we take $\mu = 4$.
- Then, we issue the same decision for all other values in the null hypothesis.

Criteria for H_0

To set H_0 we will follow these criteria, with this priority order:

1: take the null hypothesis stated in the problem;

2: if no hypothesis has been stated in [1] criteria, take the opposite of that we want to decide. E.g. : Usually mean production is on average 100. Has it decreased? So we take as the null hypothesis: $H_0 : \mu \geq 100$.

3: if nothing has been stated in [1] and [2] criteria, we take the opposite of the evidence. E.g. : Usually mean production is on average 100. $\overline{x} = 110$. It seems like production has increased, so we take $H_0 : \mu \leq 100$.

Criteria for H_0

Along the course we have taken the 3rd criteria to set the null hypothesis. Actually, most times we want to decide if the evidence is showing the truth. So, usually there is no contradiction between [2] and [3] criteria.