# Statistical testing: introduction

Josemari Sarasola

Statistics for Business

Gizapedia

# Statistical testing: introduction

## Example: Peter, the naughty (or not) student

- You are a teacher and Peter is one of your students. He took 10 at the exam. You know he isn't a good student. So you should decide if he has cheated or not.

- This kind of problems are statistical tests: you have an evidence (a fact, an event, some data) and on the basis of those data you must accept or reject a hypothesis that has been assumed.

- The starting point is always a null hypothesis: what you assume in principle, without any evidence. We name the null hypothesis $H_0$. As a general rule, the null hypotersis must be opposite of the suspicions you have, in order to be cautious. Here, it's quite clear that it seems like Peter has cheated, so you must take as $H_0$ the opposite: he has not cheated.

- The next step is calculating the probability of evidence given or assumed $H_0$, that is: $P[evidence/H_0]$. In the example, if Peter has not cheated ($H_0$), the probability of getting 10 (the evidence) is very small. We call this probability the p value.

- When the probability of something is very small, we think that is quite strange. And when the reality is strange we reject our previous assumptions. So if p-value is very small, we reject $H_0$; if not, we accept it. That's the basic logic of statistical testing.

# Statistical testing: introduction

## Steps

- Fix the null hypothesis: $H_0$ .
- We'll take into account these criteria to fix $H_0$, in this priority order:
    1. Take the hypothesis that is given.
    2. Take the opposite of that what it's asked or thought.
    3. Take the opposite of that what data are showing us.
- Calculate the probability of that what occurred (or something stranger), that is the p-value, assumed $H_0$.
- To calculate that probability take the boundary value in $H_0$.
- Compare p-value to $\alpha$ value fixed in advance.
    - $p \leq \alpha$ implies that what happened is rare, and so we reject $H_0$.
    - $p > \alpha$ implies that what happened is normal and so we accept $H_0$.

# Statistical testing: introduction

## Example: p-value

Normally, the probability of a piece being faulty is 0.15. We have bought a new machine and think that probability has decreased. Among 20 pieces, only one has been faulty. Should we conclude that the machine improves the process? $\alpha = 0.1$.

- $H_0 : p \geq 0.15$ (the opposite of that we are asking)
- We will reject $H_0 : p \geq 0.15$ when there are *few* faulty pieces among the set of 20 pieces (critical region is on the lower side).
- We take the boundary value ($p = 0.15$) as the basis.
- X: faulty pieces among the set of 20 pieces.
  $P[X \leq 1/B(n = 20, p = 0.15)] = 0.1755$.
- p-value (0.1755)is bigger than $\alpha$. So, we decide that the evidence is normal, assumed $H_0$, and accordingly we accept $H_0$.

## Statistical testing: introduction

### Example: critical value

Which is the critical value for the previous test? Critical value is
the smallest value in order to reject the null hypothesis.

- In our example, $P[X \leq 0/B(n = 20, p = 0.15)] = 0.0387$.
- So, if there had been 0 faulty pieces, we should have rejected
  the null hypothesis. As there are more faulty pieces (actually,
  1), we accept $H_0$.
- So, there are to ways to test a hypothesis:
    - Comparing p-value to $\alpha$.
    - Comparing evidence to the critical value.
- With the critical value we can design a test without having
  any evidence. But it's not possible to calculate the p-value
  without any data.