

Binomial distribution Example

• In a production process, probability of producing a faulty piece is 0.2. Pieces are produced with total independence. We produce 8 pieces. What is the probability of 3 of them being faulty?

 $P[among \ 8 \ 3X] = P[XXXOOOOO"ao"]$ = 0.2 × 0.2 × 0.2 × 0.8 × 0.8 × 0.8 × 0.8 × 0.8 × $\frac{8!}{3!5!}$ = 0.2³ × 0.8⁵ × $\frac{8!}{3!5!}$

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Binomial distribution Mass function

Binomial distribution gives us the probability of being x successes into a sequence of size n, p beigng the probability of success:

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Mass function of the binomial distribution

$$P[X = x] = p^{x} \times (1 - p)^{n - x} \times \frac{n!}{x!(n - x)!}; \quad x = 0, 1, 2, \dots, n$$

Shortly, we write it:

Binomial distribution $X \sim B(n,p)$ It has 2 parameters: *n* and *p*

Binomial distribution Example

$$0.2^3 \times 0.8^5 imes rac{8!}{3!5!}$$

• So, generally, if probability of being faulty os *p*:

$$p^3 \times (1-p)^5 \times \frac{8!}{3!5!}$$

• And if number of produced pieces is *n*:

$$p^3 \times (1-p)^{n-3} \times \frac{n!}{3!(n-3)!}$$

• And if we want to calculate the probability of being *x* faulty pieces:

$$p^{x} \times (1-p)^{n-x} \times \frac{n!}{x!(n-x)!}$$
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Binomial distribution Expected value and variance

B(n,p) distribution is the sum of $n \ b(p)$ Bernoulli distributions:

$$B(n,p) = \overbrace{b(p) + b(p) + \ldots + b(p)}^{n}$$

• Explanation (8 pieces, number of faulty pieces)

B(8,p)	b(p)							
5 faulty	0	0	0	0	0	0	0	0
	1	1	1	1	1	1	1	1

(faulty:1, faultless:0) (in red, the event that really happens)

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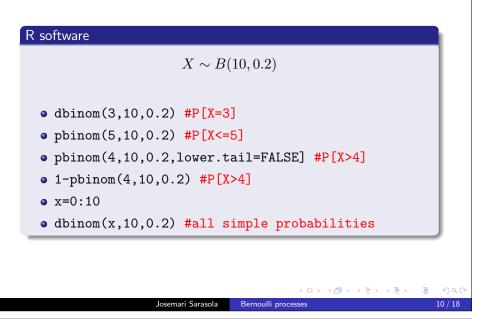
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As expected value and variance of a sum are respectively the sum of expected values and variances:

$$\mu = E[X_{B(n,p)}] = \overbrace{p+p+\ldots+p}^{n} = np$$
$$\sigma^2 = var[X_{B(n,p)}] = \overbrace{pq+pq+\ldots+pq}^{n} = npq$$

(Remember that expected value and variance of a b(p) distribution are respectively p eta pq.)

Binomial distribution



Geometric distribution

Binomial distribution gives the probability of having x successes among n trials. On the other hand, geometric distribution gives the probability of having x failures before the 1st success. Example (success: faulty: X; failure: faultless: 0):

Sequence	Faultless pieces before the first faulty piece
00X00XX	x=2
X0X00X0	x=0
000XX00	x=3

 $P[X = 3] = P[3 \text{ failures before 1st success}] = P[000X] = (1-p)^3 p$

Mass function of the geometric distribution $P[X = m] = (1 = m)^{2}m = m = 0$

 $P[X = x] = (1 - p)^{x}p; \quad x = 0, 1, 2, \dots$

• Earthquakes of magnitude 6 occur once every 1000 years. So return period of 6 magnitude eartquakes is 1000 years.

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• Fatal road accidents occur on average each 15 days in Gipuzkoa. So return period of those accidents is 15 days.

Return period (T)

Binomial distribution

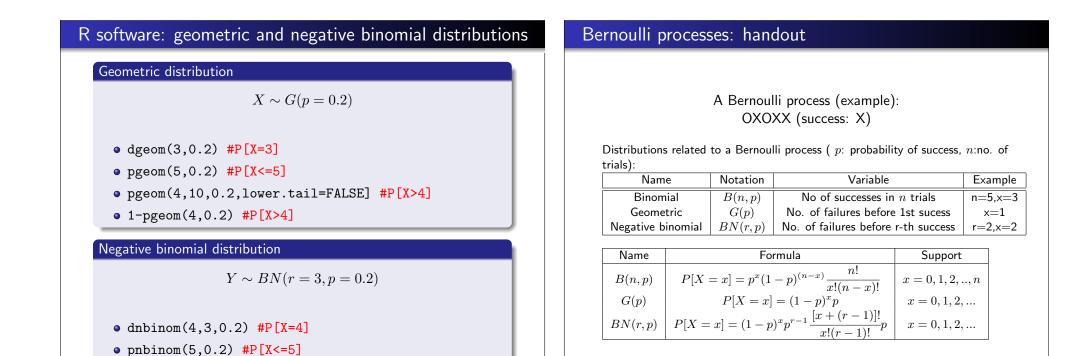
Return period

Return period is n value that gives an np expected value of 1. So, $T\times p=1 \rightarrow p=\frac{1}{T}.$

We particularly calculate return period for accidents, disasters and other events with small probability.

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Geometric distribution Negative binomial distribution Negative binomial distribution gives us the probability of having xfailures before having r successes. Notation and parameters Sequence Faultless pieces before r-th faulty piece r $X \sim G(p)$ 00X00XX... 2 x=4X0X00XX... 1 x=0It has only one parameter: p. 3 XX000XX... x=3Expected value $P[X=3]_{r=3} = P[XXOOO \text{ ao } AND X \text{ fixed}] = (1-p)^3 p^2 \frac{5!}{3!2!} p$ $E[X_{G(p)}] = q/p$ In fact, in all the following sequences it holds x = 3: $XXOOO|X | XOXOO|X | OOXXO|X | \dots$ ◆□ ▶ ◆□ ▶ ◆ □ ▶ ◆ □ ▶ ● □ ● ● ● ● ◆□ > ◆□ > ◆三 > ◆三 > ・三 ・ のへの Josemari Sarasola Bernoulli processes 13/18 Josemari Sarasola Bernoulli processes 14/18 Negative binomial distribution Negative binomial distribution Mass function Link with geometric distribution $P[X = x] = (1 - p)^{x} p^{r-1} \frac{[x + (r - 1)]!}{x!(r - 1)!} p; \quad x = 0, 1, 2, \dots$ $BN(r=1,p) \equiv G(p)$ Explanation: before *r*-th success, there must be r-1 successes and xExpected value of negative binomial distribution failures, in any order. And finally we must have a success in a fixed position. $E[X_{BN(r,p)}] = r \times q/p$ Notation and parameters $X \sim BN(r, p)$ In fact, BN(r,p) is the sum of r G(p) distributions. Think about it. It has 2 parameters: r and p ▲ロト ▲掃ト ▲ヨト ▲ヨト ニヨー のへの ▲ロト ▲圖 ト ▲ ヨト ▲ ヨト 一 ヨー つんで 16 / 18 Josemari Sarasola Bernoulli processes 15/18 Josemari Sarasola Bernoulli processes



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